Modeling The Business Cycle
Part III - Enterprise Value

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We will define enterprise value to be the present value of net cash flow expected to be received over a time interval of finite or infinite length. In this white paper we will build a model that calculates enterprise value for a company whose revenues are correlated with the business cycle. To that end we will work through the following hypothetical problem from Part I...

Our Hypothetical Problem

In Parts I and II we were tasked with forecasting revenue, net income and net investment for ABC Company. The table below presents ABC Company's go-forward model assumptions...

Table 1: Model Assumptions

Description	Balance	Notes
Annualized revenue at time zero (in thousands)	\$10,000	Current revenue annualized
Annualized revenue growth rate (%)	5.00	Discrete-time secular growth rate (RGR)
Annualized revenue volatility (%)	25.00	Secular growth rate standard deviation
Assets as a percent of annualized revenue (%)	60.00	Total assets divided by annualized revenue
Return on assets (%)	13.50	After-tax ROA
Cost of capital (%)	12.00	Discrete-time annualized discount rate
Peak-to-trough change in revenue (%)	50.00	Excludes secular growth rate
Business cycle length in months	60	Peak-to-peak or trough-to-trough

We are tasked with answering the following questions:

Question 1: What is enterprise value at time zero given that cash flow is received in perpetuity?

Question 2: Using the answer to the question above by how much do we overestimate enterprise value if we don't account for the business cycle?

Question 3: What is enterprise value at the end of year 3 given that cash flow is received over the finite time interval $[3, 20]$?

Base Equations

Table 2: Model Parameter Values From Part II

Symbol	Description	Value
R_0	Actual annualized revenue at time zero	\$10,000,000
λ	Continuous-time secular revenue growth rate	0.0488
π	After-tax return on assets	0.1350
ϵ	Ratio of total assets to annualized revenue	0.6000
β	Business cycle sine wave radians	1.2566
Δ	Business cycle sine wave amplitude	0.2500
ϕ	Current position in the business cycle (in years)	1.2500

In Part II we defined the variable A_t to be total assets at time t. Using the parameters in Table 2 above the equation for expected total assets at time t from the perspective of time zero is... [2]

$$\mathbb{E}\left[A_t\right] = \epsilon\, R_0 \left(1 + \Delta\, \sin(\beta\,\phi)\right)^{-1} \text{Exp}\left\{\lambda\, t\right\} \left(1 + \Delta\, \sin(\beta\,(t+\phi))\right) \tag{1}$$

Using Equation (1) above and the parameters in Table 2 above the equation for the derivative of total assets with respect to time is... [2]

$$\frac{\delta}{\delta t}\,\mathbb{E}\left[A_t\right] = \epsilon\, R_0 \left(1 + \Delta\, \sin(\beta\,\phi)\right)^{-1} \text{Exp}\left\{\lambda\, t\right\} \left(\lambda + \Delta\,\lambda\, \sin(\beta\,(t+\phi)) + \Delta\,\beta\, \cos(\beta\,(t+\phi))\right) \tag{2}$$

We will define the variable κ to be the continuous time discount rate and the variable α to be the difference between the secular revenue growth rate and the cost of capital. Using the model assumptions in Table 1 and the parameters in Table 2 above the equations for these two variables are...

$$\kappa = \ln(1 + 0.12) = 0.1133 \text{ ...and... } \alpha = \lambda - \kappa = 0.0488 - 0.1133 = -0.0645 \tag{3}$$

Enterprise Value

We will define the variable $\bar{N}_{a,b}$ to be the present value at time a of after-tax net income expected to be realized over the time interval $[a,b]$. Using Equations (1) and (3) above the equation for the present value of net income is...

$$\begin{aligned}
\bar{N}_{a,b} &= \int_a^b \pi\, \mathbb{E}\left[A_t\right] \text{Exp}\left\{-\kappa\,(t-a)\right\} \delta t \\
&= \pi\,\epsilon\, R_0 \left(1 + \Delta\, \sin(\beta\,\phi)\right)^{-1} \left(\int_a^b \text{Exp}\left\{\lambda\, t\right\} \delta t + \Delta \int_a^b \text{Exp}\left\{\lambda\, t\right\} \sin(\beta\,(t+\phi))\,\delta t\right) \text{Exp}\left\{-\kappa\,(t-a)\right\} \delta t \\
&= \pi\,\epsilon\, R_0 \left(1 + \Delta\, \sin(\beta\,\phi)\right)^{-1} \text{Exp}\left\{\kappa\, a\right\} \left(\int_a^b \text{Exp}\left\{\alpha\, t\right\} \delta t + \Delta \int_a^b \text{Exp}\left\{\alpha\, t\right\} \sin(\beta\,(t+\phi))\,\delta t\right)
\end{aligned} \tag{4}$$

Using Appendix Equations (16) and (17) below we can rewrite Equation (4) above as...

$$\bar{N}_{a,b} = \pi\,\epsilon\, R_0 \left(1 + \Delta\, \sin(\beta\,\phi)\right)^{-1} \text{Exp}\left\{\kappa\, a\right\} \left(I(a,b)_1 + \Delta\, I(a,b)_2\right) \tag{5}$$

We will define the variable $\bar{M}_{a,b}$ to be the present value at time a of expected cumulative investment over the time interval $[a,b]$. The equation for the present value of cumulative investment is...

$$\begin{aligned}
\bar{M}_{a,b} &= \int_a^b \frac{\delta}{\delta t}\,\mathbb{E}\left[A_t\right] \text{Exp}\left\{-\kappa\,(t-a)\right\} \delta t \\
&= \text{Exp}\left\{\kappa\, a\right\} \int_a^b \epsilon\, R_0 \left(1 + \Delta\, \sin(\beta\,\phi)\right)^{-1} \left(\lambda\, \text{Exp}\left\{\lambda\, t\right\} + \Delta\,\beta\, \text{Exp}\left\{\lambda\, t\right\} \cos(\beta\,(t+\phi))\right. \\
&\quad \left. + \Delta\,\lambda\, \text{Exp}\left\{\lambda\, t\right\} \sin(\beta\,(t+\phi))\right) \text{Exp}\left\{-\kappa\, t\right\} \delta t \\
&= \epsilon\, R_0 \left(1 + \Delta\, \sin(\beta\,\phi)\right)^{-1} \text{Exp}\left\{\kappa\, a\right\} \left(\lambda \int_a^b \text{Exp}\left\{\alpha\, t\right\} \delta t + \Delta\,\beta\lambda \int_a^b \text{Exp}\left\{\alpha\, t\right\} \cos(\beta\,(t+\phi))\,\delta t\right. \\
&\quad \left. + \Delta\,\lambda \int_a^b \text{Exp}\left\{\alpha\, t\right\} \sin(\beta\,(t+\phi))\,\delta t\right)
\end{aligned} \tag{6}$$

Using Appendix Equations (16), (17) and (18) below we can rewrite Equation (6) above as...

$$\bar{M}_{a,b} = \epsilon\, R_0 \left(1 + \Delta\,\sin(\beta\,\phi)\right)^{-1} \text{Exp}\left\{\kappa\, a\right\}\left(\lambda\, I(a,b)_1 + \Delta\,\beta\, I(a,b)_3 + \Delta\,\lambda\, I(a,b)_2\right) \tag{7}$$

We will define the variable $V_{a,b}$ to be enterprise value at time a, which is the present value of net cash flow expected to be received over the time interval $[a,b]$. Using Equations (4) and (6) above the equation for enterprise value is...

$$V_{a,b} = \bar{N}_{a,b} - \bar{M}_{a,b} \tag{8}$$

Using Equations (5) and (7) above we can rewrite Equation (8) above as...

$$V_{a,b} = \epsilon\, R_0 \left(1 + \Delta\,\sin(\beta\,\phi)\right)^{-1} \text{Exp}\left\{\kappa\, a\right\}\left((\pi - \lambda)\, I(a,b)_1 + \Delta\,(\pi - \lambda)\, I(a,b)_2 - \Delta\,\beta\, I(a,b)_3\right) \tag{9}$$

The Answers To Our Hypothetical Problem

Question 1: What is enterprise value at time zero given that cash flow is received in perpetuity?

Using Equations (3) above, the data in Table 2 above and the Appendix Equations below the values of the following integrals are...

$$I(0,\infty)_1 = 15.4946 \text{ ...and... } I(0,\infty)_2 = 0.0408 \text{ ...and... } I(0,\infty)_3 = -0.7937 \tag{10}$$

Using Equations (9) and (10) above and the data in Table 2 above the answer to the question is...

$$V_{a,b} = 0.60 \times 10,000,000 \times \left(1 + 0.25 \times \sin(1.2566 \times 1.25)\right)^{-1} \times \text{Exp}\left\{0.1133 \times 0\right\} \times \Big((0.1350 - 0.0488) \times 15.4946$$
$$+ 0.25 \times (0.1350 - 0.0488) \times 0.0408 - 0.25 \times 1.2566 \times -0.7937\Big) = 7,613,000 \tag{11}$$

Question 2: Using the answer to the question above by how much do we overestimate enterprise value if we don't account for the business cycle?

To remove cyclicallity we set the variable Δ, which is defined as the sensitivity of cash flow to the business cycle, to zero. Using Equation (11) above and setting $\Delta = 0$ enterprise value becomes...

$$V_{a,b} = 0.60 \times 10,000,000 \times \left(1 + 0 \times \sin(1.2566 \times 1.25)\right)^{-1} \times \text{Exp}\left\{0.1133 \times 0\right\} \times \Big((0.1350 - 0.0488) \times 15.4946$$
$$+ 0 \times (0.1350 - 0.0488) \times 0.0408 - 0 \times 1.2566 \times -0.7937\Big) = 8,015,000 \tag{12}$$

Question 3: What is enterprise value at the end of year 3 given that cash flow is received over the finite time interval $[3,20]$?

Using Equations (3) above, the data in Table 2 above and the Appendix Equations below the values of the following integrals are...

$$I(3,20)_1 = 8.5052 \text{ ...and... } I(3,20)_2 = 0.3460 \text{ ...and... } I(3,20)_3 = 0.7671 \tag{13}$$

Using Equations (9) and (10) above and the data in Table 2 above the answer to the question is...

$$V_{3,20} = 0.60 \times 10,000,000 \times \left(1 + 0.25 \times \sin(1.2566 \times 1.25)\right)^{-1} \times \text{Exp}\left\{0.1133 \times 3\right\}\Big((0.1350 - 0.0488) \times 8.5052$$
$$+ 0.25 \times (0.1350 - 0.0488) \times 0.3460 - 0.25 \times 1.2566 \times 0.7671\Big)$$
$$= 3,370,000 \tag{14}$$

Appendix

A. We will define the following equations... [3]

$$E_1 = \text{Exp}\left\{\alpha\, t\right\} \text{ ...and... } E_2 = \text{Exp}\left\{\alpha\, t\right\} \sin(\beta\,(t+\phi)) \text{ ...and... } E_3 = \text{Exp}\left\{\alpha\, t\right\} \cos(\beta\,(t+\phi)) \tag{15}$$

B. Using the first equation in Equation (15) above we will make the following integral definition... [3]

$$I(a,b)_1 = \int\limits_a^b E_1\,\delta t = \text{Exp}\left\{\alpha\, t\right\} \alpha^{-1} \Bigg[_a^b \tag{16}$$

C. Using the second equation in Equation (15) above we will make the following integral definition... [3]

$$I(a,b)_2 = \int\limits_a^b E_2\,\delta t = \text{Exp}\left\{\alpha\, t\right\} \bigg(\alpha\,\sin(\beta\,(t+\phi)) - \beta\,\cos(\beta\,(t+\phi))\bigg) \bigg(\alpha^2+\beta^2\bigg)^{-1} \Bigg[_a^b \tag{17}$$

D. Using the third equation in Equation (15) above we will make the following integral definition... [3]

$$I(a,b)_3 = \int\limits_a^b E_3\,\delta t = \text{Exp}\left\{\alpha\, t\right\} \bigg(\beta\,\sin(\beta\,(t+\phi)) + \alpha\,\cos(\beta\,(t+\phi))\bigg) \bigg(\alpha^2+\beta^2\bigg)^{-1} \Bigg[_a^b \tag{18}$$

References

[1] Gary Schurman, *Modeling The Business Cycle - Part I,* October, 2020.

[2] Gary Schurman, *Modeling The Business Cycle - Part II,* October, 2020.

[3] Gary Schurman, *Modeling The Business Cycle - Mathematical Supplement,* October, 2020.